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Lazy Abstraction and SAT-Based Reachability in Hardware Model Checking

Yakir Vizel*, Orna Grumberg*, Sharon Shoham†
*Computer Science Department, The Technion, Haifa, Israel
†School of Computer Science, Academic College of Tel Aviv-Yaffo

***Abstract*—In this work we present a novel lazy abstraction-refinement technique for hardware model checking, integrated with the SAT-based algorithm IC3.**

In contrast to most SAT-based model checking algorithms, IC3 avoids unrolling of the transition relation. Instead, it applies local checks, while computing over-approximated sets of reachable states. We find IC3 most suitable for *lazy abstraction*, since each one of its local checks requires different information from the checked model.

Similarly to IC3, our algorithm obtains a series of over-approximated sets of states. However, when constructing the series, different abstractions are used for different sets.

If an abstract counterexample is obtained, we either find a corresponding concrete one, or apply refinement to eliminate *all* counterexamples of the same length. Refinement makes the abstractions more precise *as* needed, and *where* needed. After refinement, the computation resumes from the same step where it was interrupted. The result is an *incremental* abstraction-refinement algorithm where the abstraction is *lazy*.

We implemented our algorithm, called L-IC3, and compared it with the original IC3 on large industrial hardware designs. We obtained significant speedups of up to two orders of magnitude.

I. Introduction

In this work we introduce a novel lazy abstraction-refinement technique for hardware model checking, integrated with the SAT-based algorithm IC3 [3].

Model checking [5] is an automatic procedure that determines whether a given system satisfies a specification. In spite of its great success in verifying hardware and software systems, the applicability of model checking is impeded by its high space and time requirements.

The introduction of SAT-based model checking algorithms [1], [15], [12], [16], [3] significantly increased the size of the verified systems. Still, the search for improved, more scalable methods is neverending.

Most SAT-based model checking algorithms are based on an unrolling of the model's transition relation in order to traverse its state space. In contrast, the recently introduced IC3 algorithm [3] avoids such unrolling. To verify a safety property, IC3 gradually builds a series of sets of states $F_0, \ldots, F_i, \ldots$, where F_i over-approximates the set of states reachable within i steps from the initial states. The computation moves back and forth along the F_i's and strengthens them by eliminating unreachable states. This is done via *local reachability checks* between consecutive sets F_i and F_{i+1}. IC3 either reaches a fixpoint, in which case all reachable states satisfy the desired property, or returns a counterexample.

Abstraction-refinement is a well known methodology for tackling the state-explosion problem. Abstraction hides model details that are not relevant for the checked property. The resulting abstract model is then smaller. *Lazy abstraction* [10], [13], developed for software model checking, is a specific type of abstraction that allows hiding different model details at different steps of the verification.

In this work we develop, for the first time, a *lazy* abstraction-refinement framework for hardware. We use the *visible variables abstraction* [11], which is particularly suitable for hardware. However, we use it in a lazy manner in the sense that different sets of visible variables are used in different iterations of the state-space traversal.

We find the IC3 algorithm most suitable for lazy abstraction since its state traversal is performed by means of local reachability checks, each involving only two consecutive sets. Thus, at each check a different set of variables is relevant.

Our model checking algorithm, called L-IC3, thus integrates a lazy abstraction-refinement mechanism into IC3. Similarly to IC3, L-IC3 computes a series of over-approximating sets F_i, each referring to a certain time frame. However, L-IC3 considers abstractions of the model during this computation. When constructing F_{i+1}, we determine a set of variables U_i, needed for its construction, and abstract both *states and transitions* accordingly. The variables in U_i are referred to as "visible", while the others are invisible and treated as inputs.

The key ingredients of L-IC3 are therefore a series Ω of over-approximating sets of states F_i and an abstraction sequence $\bar{U}$ of sets of variables U_i.

L-IC3 works in stages. Each stage consists of an *abstract model checking* step, followed by a *refinement* step. At a given stage, the abstract model checking extends both Ω and $\bar{U}$ and checks if they include a potential abstract counterexample. If not, the sequences are further extended. If a potential abstract counterexample is found, the algorithm strengthens the sets F_i by eliminating abstract states that might be a part of an abstract counterexample.

We use a nonstandard notion of abstract counterexample, based on both Ω and $\bar{U}$. It consists of a sequence of abstract states connected by abstract transitions, satisfying: (i) each transition is based on a different abstraction U_i, and (ii) each abstract state intersects the set F_i at the corresponding time frame. Our notion of counterexample reflects the incorporation of lazy abstraction into the mechanism of computing Ω.

If an abstract counterexample is found, meaning that no strengthening is possible anymore based on the abstractions, the refinement step is invoked. Refinement applies just one iteration of a *concrete* variation of IC3, on the Ω computed by the abstract model checking. By doing so, it either finds a concrete counterexample or strengthens the F_i's so that *all* concrete counterexamples of length k are eliminated. In the latter case, the U_i's are also refined by adding more visible variables to each of them, *as* needed and *where* needed. Once refinement is finished we move to the next L-IC3 stage and the abstract model checking is re-invoked, continuing the computation from iteration k+1, with the refined sequences. This makes L-IC3 *incremental*.

L-IC3 terminates with either a fixpoint, in which case we conclude that the system satisfies the property, or with a concrete counterexample.

In summary, the main contribution of our work is a novel *lazy* abstraction-refinement technique for hardware. To the best of our knowledge this is the first time lazy abstraction is considered in the context of hardware. Our abstract model checking and refinement are SAT-based. Both avoid unrolling of the transition relation. Since our framework is based on a subtle combination of the abstract and concrete models, we provide theoretical arguments to its correctness.

In order to evaluate our new algorithm we compared it with IC3 on a set of large industrial designs and properties. We obtained speedups of up to two orders of magnitude. Our experiments demonstrate that our lazy abstraction indeed uses different sets of variables in different time frames. Moreover, only a small portion of the design's variables are used.

A. Related Work

[6] and [2] suggest optimizations and extensions to IC3, but they do not combine it with a lazy abstraction-refinement mechanism ([6] suggests the use of abstraction for IC3 but without implementation details nor results). In [14], [9], [7], [4], [8], SAT-based refinement is introduced. However, they use an unrolling of the model while we use local checks a-la IC3. Similarly to [14], [4], we also exploit an unSAT-core for refinement. However, we never unroll the model, while [14] does. Further, [14] is not incremental since after refinement it resumes its (abstract) model checking from time frame 0.

IC3 [3] is sometimes also viewed as an abstraction-refinement algorithm, since it refers to over-approximated sets F_i and the strengthening of these sets resembles refinement. However, the underlying model used by IC3 is concrete, and only the concrete transition relation is considered. We, on the other hand, alternate between abstract transition relations (in the abstract model checking step) and the concrete transition relation (in the refinement step). Our algorithm thus adds a layer of abstraction-refinement on top of this over-approximation-strengthening mechanism.

II. Preliminaries

Definition 1. A *finite state transition model* is a tuple $M = (V, U, \mathit{INIT}, \mathit{TR})$ where V is a set of variables, $U \subseteq V$ is a set of state variables, $V \setminus U$ is a set of input variables, $\mathit{INIT}(V)$ is a propositional formula over V describing the initial states and $\mathit{TR}(V, V')$ is a propositional formula over V and the next-state variables $V' = \{v' \mid v \in V\}$ describing the transition relation.

We assume that $\mathit{TR}(V, V') = \bigwedge_{v \in U} (v' = f_v(V, V'))$ where $f_v(V, V')$ is a propositional formula that assigns the next value to $v \in U$ based on current and next-state variables. Note that for an input variable $v \in V \setminus U$, f_v is not defined. From this point on M is a finite state transition model.

The set of boolean variables of M induces a set of states $S = \{0, 1\}^{|V|}$, where each state $s \in S$ is given by a valuation of the variables in V. A formula over V (resp. V, V') represents the set of states (resp. pairs of states) obtained by its satisfying assignments. With abuse of notation we will refer to a formula η over V as a set of states and therefore use the notion $s \in \eta$ for states represented by η.

The formula $\eta[V \leftarrow V']$, or η' in short, is identical to η except that each variable $v \in V$ is replaced with v'.

For a formula η over $V \cup V'$ we use $\text{Vars}(\eta) \subseteq V \cup V'$ to denote all (current or next state) variables appearing in η.

Definition 2. An *over-approximated reachability sequence* (OARS) with respect to a model M and a property AGp, denoted $\Omega(M, p)$, is a sequence $\langle F_0, \ldots, F_k \rangle$ of propositional formulas over V such that the following holds:

- $F_0 = \mathit{INIT}$
- $F_i \Rightarrow F_{i+1}$ for $0 \leq i < k$
- $F_i \wedge \mathit{TR} \Rightarrow F'_{i+1}$ for $0 \leq i < k$
- $F_i \Rightarrow p$ for $0 \leq i \leq k$

The set of states represented by F_i over-approximates the states reachable from *INIT* in at most i steps. We refer to i as *time frame (or frame) i*. When M and p are clear from the context we omit them and write Ω.

Definition 3. Let Ω be an OARS. A formula η is *inductive* up to j, if $F_j \wedge \eta \wedge \mathit{TR} \Rightarrow \eta'$. η is an *invariant* up to level j if $F_i \Rightarrow \eta$ holds for each $i \leq j$.

Note that if η is inductive up to j then $F_i \wedge \eta \wedge \mathit{TR} \Rightarrow \eta'$ holds for each $i \leq j$. Due to the properties of an OARS, η is an invariant up to j iff it is inductive up to level $j - 1$, and in addition $F_0 \Rightarrow \eta$ (initialization).

A. SAT-based Reachability via IC3

IC3 [3] is a SAT-based model checking algorithm that, given a model M and a property AGp, computes increasingly long sequences $\Omega(M, p)$. The algorithm works iteratively, where at iteration k, the OARS of length k is extended to an OARS of length $k + 1$ by initializing the set F_{k+1} and possibly updating previous sets (with index $i \leq k+1$). The computation continues until either a counterexample is found or a fixpoint is reached (i.e. $F_{i+1} \Rightarrow F_i$ for some i).

One of the main features of IC3 is the fact that no unrolling of the transition relation is needed. We give a brief overview of how it operates. More details are given along the paper as needed. For the exact details we refer the reader to [3].

IC3 starts by checking if $\mathit{INIT} \wedge \neg p$ or $\mathit{INIT} \wedge \mathit{TR} \wedge \neg p'$ is satisfiable, in which case a counterexample of length zero or one is found and the algorithm terminates. If both are unsatisfiable, F_0 is initialized to *INIT* and F_1 is initialized to p. $\langle F_0, F_1\rangle$ is an OARS (it satisfies the conditions in Def. 2).

IC3 extends and updates Ω, while strengthening the F_i's. The kth iteration starts from an OARS $\langle F_0, \dots, F_k\rangle$. Then F_{k+1} is initialized to p. Clearly, $F_k \Rightarrow F_{k+1}$ and $F_{k+1} \Rightarrow p$ hold. Therefore, the purpose of strengthening is to ensure that $F_k \wedge \mathit{TR} \Rightarrow F'_{k+1}$. This is done by checking that $F_k \wedge \mathit{TR} \wedge \neg p'$ is unsatisfiable. If this formula is satisfiable then a state $s \in F_k$ is retrieved from the satisfying assignment. s is a bad state since it reaches $\neg p$ (and by that violates $F_k \wedge \mathit{TR} \Rightarrow F'_{k+1}$). At this point, either s is reachable from *INIT*, in which case a counterexample exists, or s is unreachable and needs to be removed from F_k. In order to determine if s is reachable, IC3 checks the formula: $F_{k-1} \wedge \mathit{TR} \wedge s'$. If this formula is unsatisfiable, then s can be removed from F_k (since the property $F_{k-1} \wedge \mathit{TR} \Rightarrow F'_k$ of an OARS holds without it as well), and the same process is repeated for other states in F_k that can reach $\neg p$ (if any). However, if $F_{k-1} \wedge \mathit{TR} \wedge s'$ is satisfiable, a predecessor $t \in F_{k-1}$ of s is extracted and handled similarly to s in order to determine if t (which is also a bad state) is reachable from *INIT* or not. IC3 therefore moves back and forth along the F_i's, while retrieving bad states b and checking their reachability from *INIT* via local reachability checks of the form $F_i \wedge \mathit{TR} \wedge b'$. During this process, the F_i's are strengthened by removing bad states that are not reachable[1]. If a state in $F_0 = \mathit{INIT}$ is reached during the backwards traversal, then a counterexample is obtained.

Definition 4. Satisfiability checks of the form $F_i \wedge \mathit{TR} \wedge \eta$ (where $\mathrm{Vars}(\eta) \subseteq V \cup V'$) are called *$i$-reachability checks.*

B. Abstraction

Throughout the paper we consider the "visible variables" abstraction [11]. Let $M_c = (V, U, \mathit{INIT}, \mathit{TR})$ be a model and let $U_i \subseteq U$ be a set of state-variables. We refer to U_i as the set of "visible variables".

Given U_i, we consider an abstract model $M_i = (V_i, U_i, \mathit{TR}_i)$ of M_c where $\mathit{TR}_i = \bigwedge_{v \in U_i} (v' = f_v(V, V'))$ is an abstract transition relation, and $V_i = \{v \in V \mid v \in \mathrm{Vars}(\mathit{TR}_i) \vee v' \in \mathrm{Vars}(\mathit{TR}_i)\} \subseteq V$. Note that the behavior of invisible state variables (in $U \setminus U_i$) is nondeterministic.

We do not introduce an abstraction of *INIT* as part of M_i since we always consider the concrete set of initial states. M_i is an *abstraction* of M_c, denoted $M_c \preceq M_i$, in the sense that both its set of states and its transition relation are abstractions of the concrete ones. M_i induces a set of abstract states S_i which includes all valuations to V_i. Specifically, each concrete state $s \in S$ is abstracted by the abstract state $s_i \in S_i$ that agrees with s on the assignment to the joint variables in V_i. In this case we write $s \preceq s_i$. We sometimes refer to s_i as the set of concrete states it abstracts: $\{s \in S | s \preceq s_i\}$.

[1]In fact, in order to remove a bad state b from F_i, IC3 finds a clause c that is an invariant up to i and implies $\neg b$, and adds c to F_i as a conjunct.

In addition, TR is abstracted by TR_i in the sense that $\mathit{TR} \Rightarrow \mathit{TR}_i$. Formally, the relation $\{(s, s_i) \mid s \preceq s_i\}$ is a simulation relation from M_c to M_i.

Given an OARS $\Omega(M_c, p) = \langle F_0, \dots, F_k\rangle$ and an abstract model M_i, we say that a formula η is *inductive* up to level j w.r.t. M_i, if $F_j \wedge \eta \wedge \mathit{TR}_i \Rightarrow \eta'$.

Lemma 5. *Any formula inductive up to j w.r.t. M_i is also inductive up to j w.r.t. M_c.*

The lemma holds since $\mathit{TR} \Rightarrow \mathit{TR}_i$. When we do not explicitly mention a model, we refer to inductiveness w.r.t. M_c. The notion of an invariant always refers to M_c.

C. Lazy Abstraction

As mentioned above, *lazy abstraction* [10] allows to use different details of the model at different iterations of the state-space traversal. We adapt the notion of lazy abstraction to abstraction based on *visible variables* [11], and allow different variables to be visible at different time frames.

Definition 6. An *abstraction sequence* w.r.t. a model M_c is a sequence $\bar{U} = \langle U_0, \dots, U_k\rangle$ where $U_i \subseteq U$ for $0 \leq i \leq k$, is a set of visible state-variables. $\bar{U}$ is *monotonic* if $U_i \subseteq U_{i+1}$ for each $0 \leq i < k$.

An abstraction sequence $\bar{U}$ represents different levels of abstraction of M_c. It induces a sequence of abstract models $\langle M_0, \dots, M_k\rangle$ where M_i is defined as in Sec. II-B. If $\bar{U}$ is monotonic, the induced sequence of abstract models is also monotonic in the sense that $M_0 \succeq \ldots \succeq M_k \succeq M_c$.

Definition 7. Let $\bar{U} = \langle U_0, \dots, U_k\rangle$ be a monotonic abstraction sequence and $\Omega(M_c, p) = \langle F_0, \dots, F_k\rangle$ an OARS. A sequence $s_i, \dots, s_j$ of abstract states where $0 \leq i < j \leq k+1$ is an *abstract path from i to j* if (i) for each $i \leq l < j$, $(s_l, s_{l+1}) \models \mathit{TR}_l$, and[2] (ii) for each $i \leq l \leq \min\{j, k\}$, $s_l \cap F_l \neq \emptyset$.

An abstract path $s_0, \dots, s_j$ from 0 to j is an *abstract counterexample of length j* if $s_j \cap \neg p \neq \emptyset$.

Note that the definition above is not standard. It refers to different transition relations at different steps. Also, it requires the abstract states to be part of the corresponding F_i.

Definition 8. An abstraction sequence $\langle {U_0}^r, \dots, {U_k}^r\rangle$ is a *refinement* of an abstraction sequence $\langle U_0, \dots, U_k\rangle$ if $U_i \subseteq {U_i}^r$ for each i.

III. Lazy Abstraction and IC3

In this section we describe our proposed algorithm for lazy abstraction, called L-IC3. The key ingredients of L-IC3 are an *abstraction sequence* $\bar{U}$ that induces different abstractions at different time frames as well as an *OARS* Ω.

L-IC3 starts with an initialization step and then works in *stages* (Fig. 1). Its initialization (lines 2-5) is similar to the

[2]Requirement (ii) dismisses paths that are known to be spurious based on Ω. $\min\{j, k\}$ is used for the case where $j = k + 1$, in which nonempty intersection is required only up to k.

```
1: function L-IC3($p$)
2:     $\Omega = \langle INIT, p\rangle; \bar{U} = \langle \text{Vars}(p)\rangle$
3:     if INIT-IC3($\Omega, \bar{U}, p$) == cex then
4:         return cex
5:     end if
6:     while A-IC3($\Omega, \bar{U}$) == abs-cex do
7:         if REFINE($\Omega, \bar{U}$) == cex then
8:             return cex
9:         end if
10:    end while
11:    return fixpoint
12: end function
```

Fig. 1: L-IC3

```
13: function A-IC3($\Omega, \bar{U}$)
14:     $k = |\Omega| - 1$
15:     while $\Omega.fixpoint()$ == false do
16:         $U_k = U_{k-1}$
17:         $\bar{U}.add(U_k)$
18:         $F_{k+1} = p$
19:         $\Omega.add(F_{k+1})$
20:         result = STRENGTHEN($\Omega, \bar{U}, k$)
21:         if result == abs-cex then
22:             return abs-cex
23:         end if
24:         $k = k + 1$
25:     end while
26:     return fixpoint
27: end function
```

Fig. 2: A-IC3

initialization of IC3 with one exception. If no counterexample of length 0 or 1 exists, then in addition to initializing Ω to $\langle F_0 = INIT, F_1 = p\rangle$, it initializes $\bar{U}$ to $\langle U_0 = \text{Vars}(p)\rangle$. Clearly, after initialization, Ω is an OARS.

Each L-IC3 stage (lines 6-10) consists of an abstract model checking step and a refinement step, both performed by variations of IC3. $\bar{U}$ and Ω are updated in both steps.

The abstract model checking extends and updates the OARS Ω until either a fixpoint is reached, or an abstract counterexample is found (line 6). In the latter case, the counterexample is *abstract* since it is computed w.r.t. the abstract transitions. However, it is also restricted by Ω (see Def. 7). A refinement is then performed (line 7). If the refinement finds a concrete counterexample then it terminates. Otherwise it refines $\bar{U}$ and updates Ω into an OARS (of the same length).

A new L-IC3 stage (line 6) of abstraction-refinement then begins, invoking A-IC3 with the updated Ω and the refined $\bar{U}$.

An invocation of L-IC3 results in either a fixpoint (in which case the property is proved), or a concrete counterexample.

A. Abstract Model Checking via A-IC3

The abstract model checking algorithm, A-IC3 (Fig. 2), either finds an abstract counterexample (line 22), or reaches a fixpoint (line 26) by computing an OARS Ω.

Using different abstractions The computation of Ω is done using a variation of IC3 which considers a *sequence of abstract* models, induced by a monotonic abstraction sequence $\bar{U} = \langle U_0 \dots, U_k\rangle$. Both abstract transition relations and abstract states are used. Even though abstract models are used, the obtained OARS satisfies the requirements of Def. 2, which refer to the concrete transition relation *TR*. To emphasize this, we sometimes refer to the sequence as a *concrete* OARS.

Recall that IC3 performs i-reachability checks of the form $F_i \wedge TR \wedge \eta$. A-IC3 also performs these checks (within STRENGTHEN, line 20), but instead of using the concrete *TR* it uses the *abstract* TR_i. This means that when traversing the model's state space, A-IC3 uses different abstract transition relations at different time frames. Further, when $F_i \wedge TR_i \wedge \eta$ is satisfiable, A-IC3 retrieves an *abstract* state $s_a \in M_i$ from the satisfying assignment. This abstract state is either used to strengthen Ω, or it is part of an abstract counterexample.

Incrementality If A-IC3 finds a counterexample at iteration k it returns. After refinement (line 7) A-IC3 is re-invoked with an updated Ω that is an OARS of the same length. The computation of Ω resumes from iteration $k+1$ (line 14)[3].

Iterations In iteration $k \geq 1$, the OARS $\langle F_0, \dots, F_k\rangle$ and the abstraction sequence $\langle U_0, \dots, U_{k-1}\rangle$ are extended by 1 and updated as follows (see Fig. 2).

1) Check if a fixpoint is reached. If not:
2) U_k is initialized to U_{k-1} and added to $\bar{U}$.
3) F_{k+1} is initialized to p and added to Ω.
4) The sets $F_0, \dots, F_{k+1}$ are strengthened iteratively until $\langle F_0, \dots, F_{k+1}\rangle$ becomes an OARS, or an abstract counterexample is found.

Below we describe items 2 and 4 in more detail.

(2) Extending $\bar{U}$: U_k is initialized to U_{k-1} (line 16). This is aimed at immediately eliminating from TR_k spurious transitions that lead from states in $F_{k-1} \subseteq F_k$ to $\neg p$ and were already removed from TR_{k-1}. Note that this initialization does not imply that the U_i sets will always be equal, since refinement might change them in different ways.

(4) Iterative Strengthening of Ω: A-IC3 obtains an OARS of length $k+1$ by strengthening the F_i's s.t. no abstract counterexample of length $k+1$ exists w.r.t. the OARS $\langle F_0, \dots, F_k\rangle$. This is a sufficient condition to ensure that Ω is an OARS. For this purpose, A-IC3 finds abstract states that might be a part of an abstract counterexample at a certain time frame, and attempts to block them by learning corresponding invariants. Recall that the abstract counterexamples we consider are restricted not only by the abstract transition relations, but also by the F_i sets (Def. 6). Technically, such states are described by abstract proof obligations (similarly to the notion of proof obligations used in IC3).

Definition 9. An *abstract proof obligation*, or an *obligation* in short, is a pair (s_a, n) consisting of a level $n \leq k$ and an abstract state s_a s.t. (1) s_a is a "bad state" that reaches $\neg p$ along some abstract path, (2) $\neg s_a$ is an invariant up until n, (3) $s_a \cap F_{n+1} \neq \emptyset$, and (4) F_n reaches s_a in one step of TR_n.

Thus $n+1$ is the minimal level intersecting s_a, and n is the minimal level reaching s_a in one *abstract* step. Note that it is

[3]An abstract counterexample is found w.r.t. $\Omega = \langle F_0, \dots, F_{k+1}\rangle$ produced in iteration k, where $|\Omega| = k+2$. When A-IC3 is re-invoked, k is set to $|\Omega| - 1 = k+1$.

possible that F_n cannot reach s_a along the concrete transitions. A-IC3 maintains two sets of obligations - *may* and *must*.

Definition 10. An obligation (s_a, n) is a *must obligation* w.r.t. iteration k if s_a must be shown unreachable from F_n in one step w.r.t. TR_n, in order to ensure that no abstract counterexample of length $k+1$ exists. All other obligations are *may obligations* w.r.t. k.

If s_a can reach $\neg p$ via an abstract path from level $n+1$ to level $k+1$, then (s_a, n) is a must obligation: unless s_a is blocked from F_{n+1} (by removing from F_n all states that reach s_a in one step), an abstract counterexample of length $k+1$ would exist. The same violation may also be reached from s_a in later levels F_j, $n+1 < j \leq k+1$, in which case it will be a suffix of a longer abstract counterexample with a longer prefix up to s_a. Therefore, we may also want to block s_a in F_j, $n+1 < j \leq k+1$. However, since different abstract transition relations are considered at each level, it is also possible that the same path leading from s_a to $\neg p$ is not valid from level $j > n+1$ since, for example, $U_j \supset U_{n+1}$ and hence the first transition along the path does not satisfy TR_j. The attempt to block a state s_a that is known to reach a violation from level $n+1$ in levels greater than $n+1$ creates *may* obligations[4].

The may obligations are not *required* to be blocked, but blocking them can prevent A-IC3 from encountering the same obligations/states in future iterations. On the other hand, if we report an abstract counterexample based on a may obligation, it is possible that no real abstract counterexample exists, resulting in an unnecessary refinement step which can damage the efficiency of the algorithm. We therefore greedily try to handle may obligations and strengthen Ω accordingly, but refrain from reporting abstract counterexamples based on them. Note that if a may obligation is in fact a must w.r.t. some greater k, then it will reappear as a must obligation in the following iterations.

In order to handle an obligation (s_a, n) and show s_a to be unreachable from F_n in one step, A-IC3 attempts to strengthen F_n by extracting predecessors t_a of s_a that satisfy $F_n \wedge TR_n \wedge s'_a$, defining new proof obligations based on them, and handling these obligations (by the same procedure). If F_n is successfully strengthened s.t. $F_n \wedge TR_n \wedge s'_a$ becomes unsatisfiable, then $\neg s_a$ becomes an invariant up to $n+1$.

Adding Invariants If $\neg s_a$ is an invariant up to $n+1$, then a stronger invariant that blocks s_a up to F_{n+1} is learned based on the *abstract model* M_n. Namely, $\neg s_a$ is strengthened to some sub-clause[5] c s.t. $F_0 \Rightarrow c$ and $F_n \wedge c \wedge TR_n \Rightarrow c'$, i.e. c is inductive up to n w.r.t. M_n and hence, by Lemma 5, also w.r.t. M_c. Consequently, c is also an invariant up to $n+1$, but it is a stronger invariant than $\neg s_a$ (since $c \Rightarrow \neg s_a$). The clause c is added as a conjunct to $F_0, \ldots, F_{n+1}$ while maintaining the properties of a (concrete) OARS[6].

Key procedures used by A-IC3 are described in Sec. III-D.

```
28: function REFINE($\Omega$,$\bar{U}$)
29:     result = C-STRENGTHEN($\Omega$)
30:     if result == cex then
31:         return cex
32:     end if
33:     REFINEABSTRACTION($\Omega$,$\bar{U}$)
34:     return done
35: end function
```

Fig. 3: REFINE procedure of A-IC3

B. Refinement

If A-IC3 finds an abstract counterexample of length $k+1$, refinement is invoked by L-IC3 (line 7). Refinement either finds a concrete counterexample or eliminates *all* concrete spurious counterexamples of length $k+1$. In the latter case, refinement also refines $\bar{U}$ to ensure that no *abstract* counterexample of length $k+1$ exists. Both an updated OARS $\Omega^r = \langle F_0^r, \ldots, F_{k+1}^r \rangle$ and a refined monotonic abstraction sequence $\bar{U}^r = \langle U_0^r, \ldots, U_k^r \rangle$ are returned.

The REFINE procedure is described in Fig. 3. REFINE first invokes C-STRENGTHEN, the strengthening procedure of the concrete IC3, on the sequence $\langle F_0, \ldots, F_{k+1} \rangle$ (whose prefix up to F_k is an OARS) obtained from the abstract model checking. If a concrete counterexample is found the algorithm terminates (lines 29-32). Otherwise, no concrete counterexample of length $k+1$ exists. Moreover, the updated (strengthened) sets $F_0^r, \ldots, F_{k+1}^r$ comprise an OARS. It remains to refine the abstraction sequence $\bar{U}$ in order to eliminate all *abstract* counterexamples of length $k+1$ as well. Thus, REFINEABSTRACTION is invoked (line 33).

REFINEABSTRACTION*:* A-IC3 found an abstract counterexample since it failed to strengthen the F_i's. Meaning, the relevant i-reachability checks $F_i \wedge TR_i \wedge t'_a$ could not be made unsatisfiable when using TR_i. C-STRENGTHEN, on the other hand, succeeds to do so. Namely, for each i-satisfiability check $F_i \wedge TR_i \wedge t'_a$ of A-IC3 that was satisfiable, C-STRENGTHEN manages to make the corresponding check $F_i^r \wedge TR \wedge t'$ for each $t \preceq t_a$ unsatisfiable, either by strengthening F_i^r or simply since it considers TR. Moreover, once $F_i^r \wedge TR \wedge t'$ becomes unsatisfiable, C-STRENGTHEN derives from it a clause $c \Rightarrow \neg t$ s.t. $F_i^r \wedge c \wedge TR \Rightarrow c'$ holds. C-STRENGTHEN strengthens Ω^r by adding c (invariant) as a new clause in all sets up to F_{i+1}^r. We consider it a *learned clause* at level $i+1$. The purpose of REFINEABSTRACTION is to ensure that for a learned clause c at level $i+1$, $F_i^r \wedge c \wedge TR_i^r \Rightarrow c'$ (with TR_i^r instead of TR) also holds. Meaning, c is inductive up to i w.r.t. M_i^r.

Lemma 11. *Let c be a clause learned by* C-STRENGTHEN *at level $i+1$. If $F_i^r \wedge \mathrm{TR}_i^r \Rightarrow {F_{i+1}^r}'$ then $F_i^r \wedge c \wedge \mathrm{TR}_i^r \Rightarrow c'$.*

Based on the previous lemma, in order to ensure $F_i^r \wedge c \wedge TR_i^r \Rightarrow c'$, it suffices to ensure unsatisfiability of $F_i^r \wedge TR_i^r \wedge \neg {F_{i+1}^r}'$ for every level $i+1$ in which learned clauses exist.

[4]IC3 does not make a distinction between may and must obligations and handles them all the same since in the concrete case, a longer counterexample is always a *valid* path (its suffix reaching a violation is always valid).

[5]A state s_a is represented by a conjunction of literals, which makes its negation $\neg s_a$ a clause (i.e., a disjunction of literals). A sub-clause of $\neg s_a$ consists of a subset of its literals.

[6]c is not necessarily inductive w.r.t. M_i where $i < n$ (in case $U_i \subset U_n$).

To ensure unsatisfiability of a formula $F_i^r \wedge TR_i^r \wedge \neg {F_{i+1}^r}'$, we consider the same formula over *TR*, which is clearly unsatisfiable. We derive from it an unSAT-core. The next-state variables that appear in the unSAT-core, denoted $NS(\text{unSatCore}) = \{v \in V \mid v' \in \text{Vars}(\text{UnSatCore})\}$, are added to U_i.

Lemma 12. *Let $F_i^r \wedge \text{TR} \wedge \eta'$ be an unsatisfiable formula and let UnSatCore be its unsat core. Let $U_i^r \supseteq NS(\mathit{UnSatCore})$. Then $F_i^r \wedge \text{TR}_i^r \wedge \eta'$ is unsatisfiable.*

Finally, we propagate variables that were added to U_i^r forward in order to obtain a monotonic abstraction sequence. Since we only add variables to U_i^r, i.e. make the transition relation TR_i^r more precise, then the corresponding formulas remain unsatisfiable.

C. Correctness Arguments

The OARS obtained by L-IC3 is concrete. Specifically, it does not necessarily satisfy $F_i \wedge TR_i \Rightarrow F_{i+1}$. This results both from refinement that adds invariants learned based on the concrete *TR*, and from A-IC3 that learns an invariant based on some TR_i, but also adds it to F_{j+1} for $j < i$ even if it is not inductive w.r.t. TR_j. This complicates the correctness proof.

In particular, in IC3, when a proof obligation (s, n) is handled, then for any predecessor t of s, $\neg t$ is an invariant up to $n-1$, otherwise s would belong to a lower frame (since $F_i \wedge TR \Rightarrow F_{i+1}$). Now consider an abstract proof obligation (s_a, n). If we assume to the contrary that the predecessor t_a intersect some F_i (for $i < n$) then we can still deduce that the transition $(t_a, s_a) \models TR_n$ also exists at a lower frame, i.e. $(t_a, s_a) \models TR_i$ for $i < n$. This is since $TR_n \Rightarrow TR_i$ (recall that the same does not necessarily hold for $i > n$). However, if $t_a \cap F_i \neq \emptyset$, we cannot immediately deduce that $s_a \cap F_{i+1} \neq \emptyset$ since $F_i \wedge TR_i \Rightarrow F_{i+1}$ might *not* hold. It turns out that this property does hold (see Lemma 15), but more complicated arguments are needed, based on the following:

Lemma 13. *Let $\Omega = \langle F_0, \dots, F_{k+1}\rangle$ and $\bar{U} = \langle U_0, \dots, U_k\rangle$ be the sequences obtained at the end of a refinement step or at the end of an iteration of A-IC3 in the case that no counterexample was found. Then*

1) *Ω is an OARS.*
2) *For every clause c that was added to some F_i in Ω there exists some $j \geq i-1$ s.t. c is inductive up to j w.r.t. M_j.*
3) *No abstract counterexample of length $k+1$ exists w.r.t. the prefix $\langle F_0, \dots, F_k\rangle$ of Ω.*

Theorem 14. *L-IC3 either terminates with a fixpoint, in which case the property holds, or with a concrete counterexample.*

D. Detailed Description of Strengthening

We now describe the procedures used by A-IC3 in detail.

STRENGTHEN *(Fig. 4)*: STRENGTHEN starts by checking $F_k \wedge TR_k \wedge \neg p'$ (line 37). If it is unsatisfiable, then $F_k \wedge TR \wedge \neg p'$ is also unsatisfiable as well (since $TR \Rightarrow TR_k$). Thus Ω is already an OARS and no further strengthening is needed.

Assume $F_k \wedge TR_k \wedge \neg p'$ is satisfiable. An abstract state $s_a \in M_k$ that reaches $\neg p$ in one abstract step is extracted from the satisfying assignment, meaning $s_a \cap F_k \neq \emptyset$. All concrete states in $s_a \cap F_k$ can reach $\neg p$ via TR_k and therefore, if the property is to be proven, s_a must be blocked in F_k. Otherwise, an abstract counterexample exists.

```
36: function STRENGTHEN($\Omega$,$\bar{U}$,$k$)
37:     while $F_k \wedge TR_k \wedge \neg p' == SAT$ do
38:         $obligations = \emptyset$
39:         retrieve abstract predecessor $s_k$
40:         if BLOCKSTATE($\Omega$,$s_k$,$k$,$k$,must) == abs-cex then
41:             return abs-cex
42:         end if
43:         while $obligations \neq \emptyset$ do
44:             $((s_a, n), handleMay)$ = CHOOSENEXT($obligations$)
45:             if $F_n \wedge TR_n \wedge s_a'$ == SAT then
46:                 retrieve abstract predecessor $t_n$
47:                 if BLOCKSTATE($\Omega$,$t_n$,$n$,$k$,must) == abs-cex then
48:                     if $handleMay$ then
49:                         $obligations.clearAllMust()$
50:                     else
51:                         return abs-cex
52:                     end if
53:                 end if
54:             else
55:                 $obligations.removeMust(s_a,n)$
56:                 BLOCKSTATE($\Omega$,$s_a$,$n+2$,$k$,may)
57:             end if
58:         end while
59:     end while
60:     PROPAGATECLAUSES($\Omega$)
61:     return done
62: end function
```

Fig. 4: Iterative strengthening of A-IC3

In order to block s_a in F_k, STRENGTHEN calls BLOCKSTATE on the bad state s_a at level k (line 40). BLOCKSTATE either finds a counterexample or initializes the set(s) of obligations to reflect the need to block s_a (and possibly adds invariants to the F_i's).

STRENGTHEN then handles the proof obligations one at a time. CHOOSENEXT (line 44) first considers obligations from the must set only. Obligations are chosen in increasing order of their time frames. If the must set becomes empty, then as long as the may set is not empty, one may obligation with a minimal time frame is moved from the may set to the must set. STRENGTHEN then continues, with the exception that counterexamples are no longer reported.

Given a proof obligation (s_a, n):

- If F_n can indeed reach s_a in one (abstract) step, i.e., $F_n \wedge TR_n \wedge s_a'$ is satisfiable, then a predecessor t_a of s_a s.t. $t_a \cap F_n \neq \emptyset$ is extracted from the satisfying assignment (line 46). By Lemma 15, $t_a \cap F_i = \emptyset$ for all $i < n$. Thus $\neg t_a$ is an invariant up to $n-1$. Next, the state t_a needs to be blocked (eliminated) from level $l = n$ (line 47).
- When $F_n \wedge TR_n \wedge s_a'$ becomes unsatisfiable, the proof obligation (s_a, n) is removed (line 55) since s_a can no longer be reached from level n. In fact, $\neg s_a$ is now an invariant up to level $n+1$. In order not to encounter s_a in later iterations, we speculatively attempt to block (eliminate) s_a from level $l = n+2$, while using the may parameter (line 56).

A counterexample found by BLOCKSTATE is reported iff may obligations are not yet handled (lines 41 and 51).

```
63: function BlockState($\Omega$,$t_a$,$l$,$k$,$type$)
64:     if $l > k + 1$ then
65:         $min = k + 1$
66:     else
67:         $min$ = FindNonInductive($\Omega$,$\neg t_a$,$l - 1$,$k$)
68:         if $min == 0$ then
69:             return abs-cex
70:         end if
71:         if $min \leq k$ then
72:             if type == must && min == l-1 then
73:                 $obligations.addMust(t_a, min)$
74:             else
75:                 $obligations.addMay(t_a, min)$
76:             end if
77:         end if
78:     end if
79:     AddInvariant($\Omega$,$\neg t_a$,$min$)
80:     return done
81: end function
```

Fig. 5: BLOCKSTATE procedure of A-IC3

Lemma 15. *Let (s_a, n) be a proof obligation, and let t_a be an abstract state such that $(t_a, s_a) \models \mathrm{TR}_n$. Then $t_a \cap F_i = \emptyset$ for every $i \leq n - 1$.*

BLOCKSTATE *(Fig. 5):* BLOCKSTATE(Ω,t_a,l,k,$type$) is used for blocking a "bad state" t_a from level l up to $k + 1$, where $\neg t_a$ is already known to be an invariant up to $l - 1$.

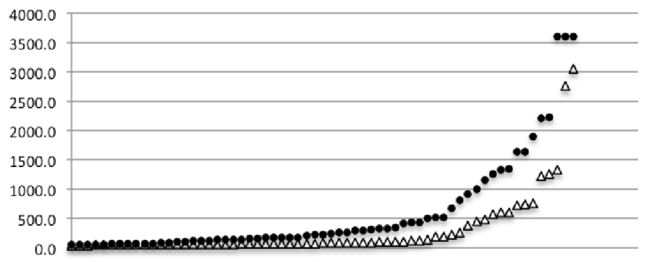
Note that if $l > k + 1$ (line 65) then t_a is already blocked up to $k + 1$. Thus $\neg t_a$ is added as an invariant up to $k + 1$ (line 79). Otherwise, BLOCKSTATE looks for a level such that $\neg t_a$ is invariant up to it.

Specifically, BLOCKSTATE looks for the minimal level min between $l - 1$ and k s.t. $F_{min} \wedge \mathit{TR}_{min} \wedge t'_a$ is satisfiable (line 67). The important property is that $\neg t_a$ is an invariant up to min: If $min = l - 1$, this holds since $\neg t_a$ is already known to be an invariant up to level $l - 1$ (this is also why the search for min starts at $l - 1$). If $min > l - 1$, then the fact that $F_{min-1} \wedge \mathit{TR}_{min-1} \wedge t'_a$ is unsatisfiable implies that $\neg t_a$ is inductive at $min - 1$ w.r.t. M_{min-1}, and hence, by Lemma 5 also w.r.t. M_c. Thus, it is an invariant up to min.

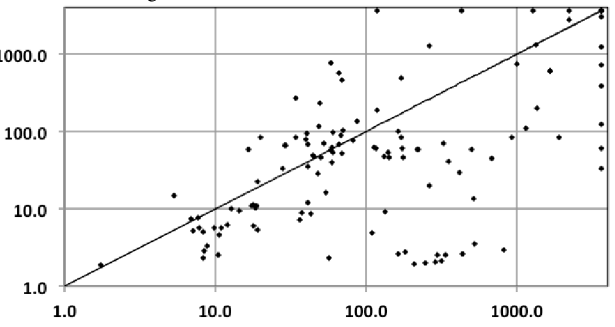
If $min = 0$, then the "bad state" t_a is reachable from *INIT* in one step of TR_0. Thus, an abstract counterexample is reported (line 69). If $min = k + 1$ then no corresponding level was found up to k, i.e., $\neg t_a$ is an invariant up to $k + 1$ and no new proof obligation is added. However, if $min \leq k$ is found then the pair (t, min) is added as a new proof obligation (lines 72-76). Either way, $\neg t_a$ is added as an invariant up to min by calling ADDINVARIANT (line 79). ADDINVARIANT learns an invariant that strengthens $\neg t_a$ and adds it to $F_0, \ldots, F_{min}$.

Classifying obligations as may/must is performed in lines 72- 76 of BLOCKSTATE. Note that only obligations of the form $(t_a, l - 1)$ are must obligations.

PROPAGATECLAUSES*:* Similarly to IC3, if the main loop in STRENGTHEN terminates, added clauses are propagated forward by PROPAGATECLAUSES (line 60). Specifically, if $F_i \wedge c \wedge \mathit{TR}_i \wedge \neg c'$ is unsatisfiable then the clause c from F_i can safely be added to F_{i+1} while maintaining the properties of an OARS. This is done in order to get to a fixpoint.



(a) Runtime trend. Dots represent IC3, triangles represent L-IC3. Test-cases are sorted in an increasing runtime order.



(b) Comparing runtime. IC3 on X-axis and L-IC3 on Y-axis

Fig. 6: Runtime information for L-IC3 and IC3

IV. EXPERIMENTAL RESULTS

For the implementation of the two algorithms we collaborated with *Jasper Design Automation*[7]. We used Jasper's formal verification platform in order to implement both the original IC3 and our L-IC3 algorithm. In both implementations we used optimizations from [6] (such as ternary simulation). Implementing these algorithms using Jasper's platform allowed us to develop and experiment with various real-life industrial designs and properties from various major semi-conductor companies. All designs contain thousands of state variables in the cone of influence of the properties.

The timeout was set to 3600 seconds and experiments were conducted on systems with Intel Xeon X5660 running at 2.8GHz and 24GB of main memory.

We experimented with 122 real safety properties from different designs. Fig. 6 shows two different analyses comparing the runtime of L-IC3 and IC3. Runtime trends are shown in Fig. 6a. As can be seen, the overall trend is in favor of L-IC3. In Fig. 6b runtime for IC3 and L-IC3 is represented by the X-axis and Y-axis respectively. We can clearly see the advantage of using L-IC3 on the more complicated test cases. These test cases are represented by the dots that are below the diagonal by a big margin. On these examples, the improvement in runtime is up to ***two*** orders of magnitude. The cases where IC3 performs better are usually cases where L-IC3 spends most of the time in refinement. Also, for false properties (counterexample exists), the performance of L-IC3 is affected by the way we treat *may* and *must* obligations. Due to our special handling, L-IC3 may lose the ability to

[7]An EDA company: http://www.jasper-da.com

N	♯Vars	Laziness - Time Frames and Number of Vars																	
		♯TF	♯AV	♯TF	♯AV	♯TF	♯AV	♯TF	♯AV	♯TF	♯AV	♯TF	♯AV	♯TF	♯AV	♯TF	♯AV	♯TF	♯AV
f_1	11866	[0-0]	323	[1-1]	647	[2-2]	686	[3-3]	699	[4-4]	705	[5-5]	713	[6-6]	714	[7-7]	728	[8-8]	743
		[9-9]	752	[10-10]	755	[11-11]	761	[12-12]	767	[13-13]	777	[14-14]	783	[15-15]	789	[16-18]	811		
f_2	5693	[0-7]	12																
f_3	5693	[0-0]	8	[1-1]	56	[2-2]	64	[3-3]	74	[4-4]	82	[5-7]	91						
f_4	5693	[0-6]	31	[7-7]	42	[8-8]	51	[9-13]	54										
f_5	5773	[0-0]	260	[1-1]	381	[2-2]	401	[3-3]	419	[4-34]	430								
f_6	1183	[0-0]	185	[1-1]	248	[2-2]	255	[3-3]	259	[4-4]	262	[5-5]	268	[6-8]	270	[9-9]	273	[10-30]	274
f_7	1247	[0-0]	57	[1-1]	62	[2-2]	73	[3-7]	76										
f_8	1247	[0-0]	63	[1-1]	64	[2-2]	72	[3-6]	83										
f_9	1277	[0-0]	263	[1-1]	303	[2-2]	318	[3-3]	321	[4-4]	322	[5-5]	323	[6-26]	347				
f_{10}	1389	[0-0]	253	[1-1]	304	[2-2]	324	[3-3]	341	[4-4]	351	[5-5]	355	[6-7]	363	[8-9]	399	[10-10]	409
		[11-12]	415	[13-13]	419	[14-16]	429	[17-18]	431										
f_{11}	1183	[0-0]	79	[1-1]	113	[2-9]	114												
f_{12}	1204	[0-0]	58	[1-1]	67	[2-2]	75	[3-7]	76										
f_{13}	3844	[0-0]	470	[1-1]	504	[2-2]	528	[3-3]	533	[4-4]	534	[5-11]	650						
f_{14}	3832	[0-0]	333	[1-1]	365	[2-2]	386	[3-5]	391	[6-6]	442	[7-10]	446						
f_{15}	3854	[0-0]	428	[1-1]	453	[2-2]	495	[3-3]	499	[4-4]	503	[5-5]	560	[6-6]	574	[7-7]	576	[8-10]	577
f_{16}	3848	[0-0]	432	[1-1]	462	[2-2]	487	[3-3]	498	[4-4]	501	[5-5]	634	[6-6]	650	[7-13]	658		
f_{17}	3854	[0-0]	426	[1-1]	480	[2-2]	525	[3-3]	539	[4-4]	540	[5-5]	559	[6-11]	570				
f_{18}	3848	[0-0]	469	[1-1]	547	[2-2]	551	[3-3]	553	[4-4]	635	[5-5]	672	[6-10]	674				

TABLE I: Lazy abstraction. N stands for the name of the verified property. ♯*Vars* stands for the number of state variables in the concrete model M_c. ♯TF stands for the time frames and ♯AV represents the number of variables (defining the abstract TR_i) in the abstract model M_i at the given time frame i (appearing in the column ♯TF).

find a counterexample which is longer than the length of the computed Ω. In those cases, IC3 may perform better. Note that the scatter at the middle is a bunch of comparable properties where both algorithms are on par.

In the given timeout, 7 properties cannot be solved by IC3 but are solved by L-IC3; 5 properties cannot be solved by L-IC3 but are solved by IC3. There are also 5 properties that cannot be solved by either algorithm. The overall runtime for IC3 is 75558 seconds while for L-IC3 it is 55424 seconds.

The laziness of our abstraction-refinement algorithm is demonstrated in Table I. The table shows how the abstraction is refined along increasing time frames. Different frames contain different variables that are needed in order to prove or disprove the given property. This demonstrates the fact that L-IC3 indeed takes advantage of the lazy abstraction framework.

Table II presents runtime characteristics for L-IC3 and IC3. In particular, it shows the number of clauses and the number of variables in Ω when either a fixpoint or a counterexample is found. In many of the examples the number of clauses produced by L-IC3 for its Ω is significantly smaller than the number of clauses produced by IC3. Recall that each of the clauses is learned via several local reachability checks. The reduced number of clauses thus indicates that L-IC3 applies a smaller number of checks and therefore issues a smaller number of calls to the SAT solver. This can explain the speedups it obtains.

An additional reason for the speedups is the fact that the local reachability checks of L-IC3 are easier than those of IC3. This is because the abstract transition relations TR_i are much smaller (in number of variables) than TR (see table I). Further, the sets F_i, computed by L-IC3 are smaller than those computed by IC3 (see Table II).

Recall that in Section III-A we distinguish between *must* and *may* obligations. The results reported above are obtained while using this distinction and handling all the *may* obligations after the *must* obligations, as described there. We also tried

N	♯Vars	Stat	♯V[Ω]	♯V[Ω_L]	♯C[Ω]	♯C[Ω_L]	k	k_L	T	T$_L$
f_1	11866	false	1001	**818**	8457	**3939**	15	18	1646	**599**
f_2	5693	true	236	**11**	617	**62**	14	8	133	**9.2**
f_3	5693	true	229	**121**	1314	**570**	13	8	351	**40.5**
f_4	5693	true	104	**24**	2101	**32**	32	14	513	**13.6**
f_5	5773	true	$>$ 616*	**414**	$>$ 16689*	**12425**	7*	35	TO	1223
f_6	1183	true	432	**370**	50511	**29316**	36	31	2216	2763
f_7	1247	true	250	**152**	10732	**238**	11	8	432	**2.6**
f_8	1247	true	177	**96**	14702	**293**	8	7	520	**3.5**
f_9	1277	false	357	**331**	8762	**3788**	13	27	164	**101**
f_{10}	1389	false	**397**	417	**12455**	19742	13	19	**262**	1268
f_{11}	1183	true	114	**106**	29183	**2589**	9	10	1153	**109**
f_{12}	1204	true	114	**105**	18698	**229**	8	8	818	**3.0**
f_{13}	3844	true	**320**	578	**547**	1529	10	12	**16.7**	59.1
f_{14}	3832	true	650	**488**	2414	**1553**	12	11	117	**61**
f_{15}	3854	true	$>$ 470*	**666**	$>$ 8320*	**5363**	6*	11	TO	**730**
f_{16}	3848	true	$>$ 687*	**826**	$>$ 7733*	**5506**	8*	14	TO	**381**
f_{17}	3854	true	811	**673**	10934	**1837**	13	12	919	**83**
f_{18}	3848	true	898	**716**	9889	**2080**	13	11	1891	**84**
f_{19}	3848	true	**966**	$>$ 216*	**13370**	$>$ 266*	11	7*	**2225**	TO

TABLE II: Running parameters for various properties. *N* stands for the name of the verified property. ♯*Vars* stands for the number of state variables in the cone of influence. ♯V[Ω] - number of variables in Ω, ♯C[Ω] - number of clauses in Ω, k - size of $\Omega(M, p)$ and T - the runtime in seconds. The subscript L represents the value for the *Lazy* version (L-IC3).

other configurations. For example, we ran experiments that do not distinguish between *must* and *may* obligations. Our experiments show that distinguishing between the two yields a better overall performance.

In addition to the industrial experiments, we also ran experiments on the HWMCC'11 benchmark. We used the test-cases with single properties. Most of the properties in this benchmark are fairly easy and can be solved in a matter of a few seconds both by IC3 and L-IC3. There are also a few cases where IC3 performs better or even reaches a result while L-IC3 does not. In these cases L-IC3 spends most of the time in refinement. On the other hand, there are several test cases that can only be solved by L-IC3 while IC3 reaches timeout.

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